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A REVIEW OF SIMPLE ENTRAINMENT CALCULATION METHODS FOR THE THERMAL SPILL PLUME

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ABSTRACT

The design of smoke management systems for buildings such as atria, covered shopping malls and sports arenas require appropriate calculation methods to predict the volume of smoky gases produced in the event of a fire. The volume of smoke must be calculated in order to determine the required fan capacity or ventilator area for a smoke management system.

In design, consideration is often given to entrainment of air into a smoke flow from a compartment opening that subsequently spills and rises into an adjacent atrium void. This type of plume is commonly known as a thermal spill plume. This article presents a literature review of existing empirically based simplified formulae to predict entrainment for the spill plume. The available methods for both a balcony and an adhered spill plume are given and the basis behind the derivation of these methods is presented and discussed.

1. INTRODUCTION

Smoke management design requires appropriate entrainment calculation methods to predict the volume of smoky gases produced in a fire in order to determine the required exhaust fan capacity or ventilator area for a design clear layer height. There are various methods available for the thermal spill plume, which include a range of empirical simplified design formulae [1-6], more complex analytical methods [7,8] and the use of Computational Fluid Dynamics (CFD) modelling.

Simplified spill plume formulae typically consist of a single line equation to predict the mass flow rate of gases produced by a spill plume. These formulae are relatively quick to use and ease the task of designing a smoke management system. They are commonly used and are given within international guidance documents such as NFPA 92B [1] and the fire engineering guide produced by the Chartered Institution of Building Services Engineers (CIBSE), UK [2]. These simplified formulae are empirically based and have specific limitations depending on the way the correlation was derived. Therefore, they generally apply for relatively idealised designs in line with the experiments from which they are based. These formulae are particularly useful for the early stages of design and for approving authorities as an initial assessment of calculations based on more complex methods.

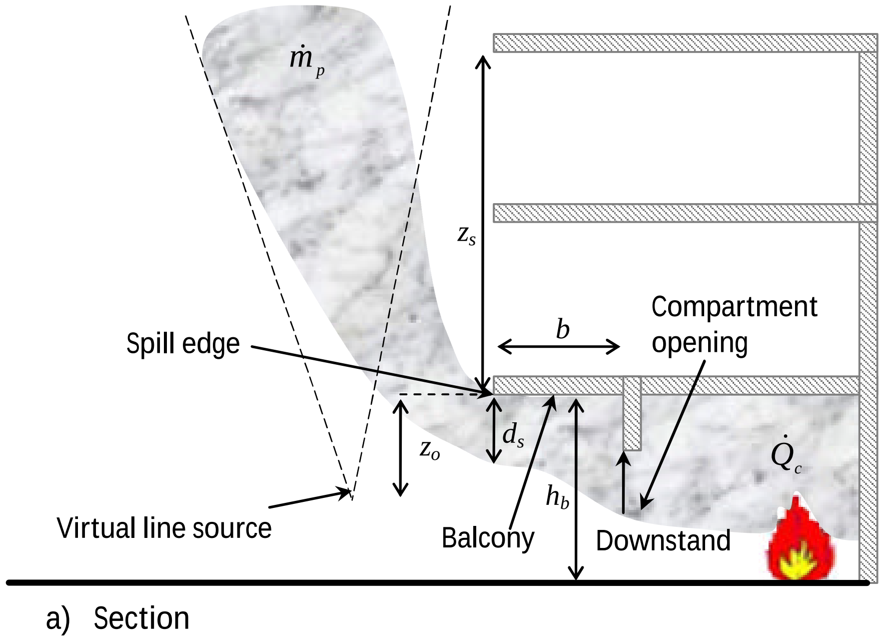
The designer also has the option of using more complex analytical methods to calculate entrainment for a spill plume. In general, analytical methods are less user-friendly compared to simple empirical formulae, typically requiring the designer to program the method into a computer. However, once this is successfully done, a result can quickly be achieved. All these methods have at least one empirical element that is required to achieve a result (i.e. an entrainment coefficient) similar to that used for entrainment into axisymmetric plumes given by Morton et al. [9]. Therefore, these methods generally apply to relatively idealised designs in line with the experiments from which the entrainment coefficient was determined.

Given the number of simple formulae available, the history of their development and some recent work, it is a useful point in time to review of the existing simple methods for the thermal spill plume. This paper does not include a review of analytical calculation methods or CFD modelling of the spill plume as these topics likely need separate papers in themselves.

2. SIMPLIFIED DESIGN FORMULAE FOR THE BALCONY SPILL PLUME

Fig. 1 shows a schematic drawing of a balcony spill plume. In the event of a fire, smoke will flow from the compartment opening and then under a horizontal projection which may extend beyond the opening (e.g. a balcony). If there is a downstand at the compartment opening, then the smoke flow will rise from the opening and form a flowing layer beneath the balcony. The smoke layer below the balcony edge will then rotate at the spill edge (i.e. at the edge of the balcony) and rise as an unhindered plume in the atrium void entraining air into both sides of plume and into the ends if they are free. Spill plumes which do not include entrainment into the ends of the plume are known as two-dimensional (2-D) plumes and those which include end entrainment are known as three-dimensional (3-D) plumes. If the smoke flow from the compartment opening is allowed to pass unrestricted under a balcony, it will spread laterally. The smoke flow will spill at the balcony edge and rise into the atrium space as a wide spill plume with a large surface area over which entrainment of air occurs. The amount of entrainment can be reduced by restricting the ability of the smoke flow to spread laterally with the use of channelling screens (otherwise referred to as draft curtains) beneath the balcony to 'channel' the flow to the balcony edge.

The majority of simplified spill plume formulae have been developed using the assumption that the plume is generated from a line plume with a virtual source of zero width located below the spill edge. Most simple formulae make the fundamental assumption of similarity between horizontal cross sectional distributions of velocity and temperature across the plume. These distributions are assumed to be Gaussian in nature. A constant empirical entrainment coefficient is also assumed to apply over the full height of rise of the plume.



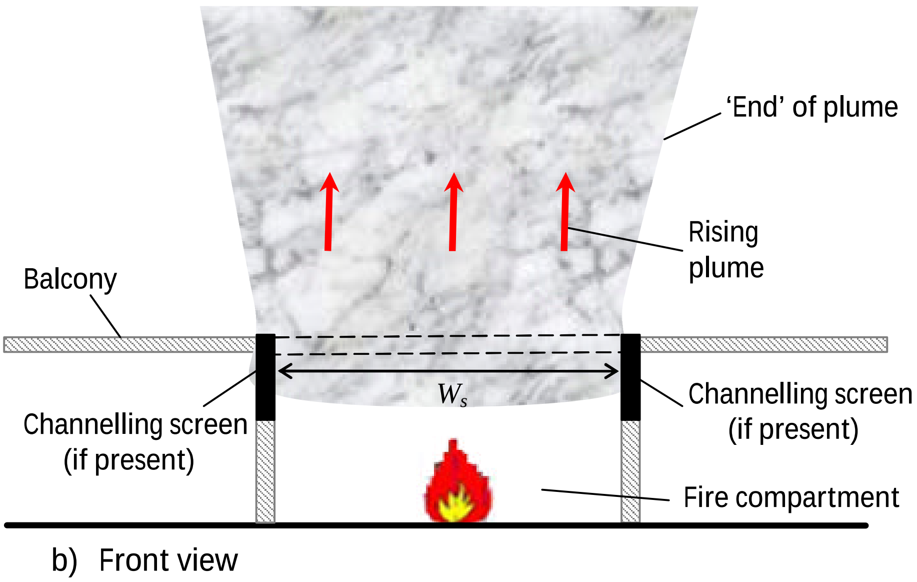


Fig. 1: A balcony spill plume

2.1 Entrainment of Air into a Thermal Line Plume

The behaviour of line plumes was first studied both experimentally and theoretically by Lee and Emmons [10]. The plumes produced were 2-D in nature, with full height channelling screens being used to prevent entrainment into the ends of the plume. The plumes were characterised by measuring horizontal temperature profiles with respect to height. The theoretical analysis assumed that velocity and temperature profiles across the horizontal cross-section of the plume were Gaussian in nature and self similar (i.e. Gaussian profiles remain Gaussian with height). The theory employs a constant empirical entrainment coefficient, a , over the height of rise of the plume (as per Morton et al. [9] for axisymmetric plumes), where entrainment into the plume is proportional to the centreline velocity.

The Lee and Emmons model for the mass flow rate per unit width in the plume is given by:

$$\frac{\dot{m}_p}{W} = C_m \left(\frac{r_{amb}^2 g \dot{Q}_c}{c_{p,air} W T_{amb}} \right)^{1/3} \left(z + z_0 \right) \tag{1}$$

where $C_m = \sqrt{p}\, a^{2/3} \left(1 + l^2\right)^{1/6}$ and it is assumed that the Boussinesq approximation applies (i.e. the plume temperature and density are both equal to ambient values in the mass and momentum terms, but not in the buoyancy term).

Lee and Emmons empirically determined values of the universal constants in equation (1), with a = 0.16 and l = 0.9, which gives rise to C_m = 0.58. The mass flow rate of gases in the plume is given by:

$$\dot{m}_p = C_m \left(\frac{r_{amb}^2 g}{c_{p,air} T_{amb}} \right)^{1/3} \dot{Q}_c^{1/3} W^{2/3} \left(z + z_0 \right) \tag{2}$$

Typical values of r_{amb}, g, $c_{p,air}$ and T_{amb} gives a value of $\left(\frac{r_{amb}^2 g}{c_{p,air} T_{amb}} \right)^{1/3}$ which is approximately constant at 0.36. Therefore it is convenient and common to see equation (2) in the following form:

$$\dot{m}_p = C \dot{Q}_c^{1/3} W^{2/3} \left(z + z_0 \right) \tag{3}$$

where, $C = C_m \left(\frac{r_{amb}^2 g}{c_{p,air} T_{amb}} \right)^{1/3}$

There is some variation in the proposed values of C_m and a in the literature from more recent line plume studies [11]. Poreh et al. [3] note that differences in the proposed values of C_m and a depended on whether estimates were based on direct measurements of the mass flow rate or calculated from velocity and temperature distributions. From the study of 2-D balcony spill plumes, Poreh et al. [3] determined the value of C_m = 0.44 with a = 0.11. This was lower than the value of a = 0.16 proposed by Lee and Emmons [10]. In general, there is increasing evidence to suggest a reduced value of a from that originally proposed by Lee and Emmons.

2.2 Law (1986)

Law [4] developed a simplified spill plume formula by modifying a relationship developed by Yokoi [12] between the maximum gas temperature rise and the total heat release rate of the fire for flows from a window such that:

$$q_{\max} = \frac{6.89}{\left(z_s + z_0 \right)} \left(\frac{\dot{Q}_t}{W_s} \right)^{2/3} \tag{4}$$

Law used data from Morgan and Marshall [13] to correlate the maximum temperature rise in the spill plume, $q_{max,p}$ versus $\left(\dot{Q}_t / W_s \right)$ at two specific heights of rise of plume. Law also correlated the maximum temperature rise in the gas layer below the spill edge, $q_{max,s}$ versus $\left(\dot{Q}_t / W_s \right)$. These data provided relationships which obeyed a 2/3 power law as proposed by Yokoi. Law used these correlations to estimate the location of the virtual line source below the spill edge (i.e. z_0). When correlating $q_{max,p}$ the virtual source was estimated to be a distance of $0.67h_b$ below the spill edge. However, when $q_{max,s}$ was correlated the virtual source was estimated to be a distance of $0.5h_b$ below the spill edge.

Law used $q_{max,s}$ and the conservation of energy to develop a relationship for the mass flow rate of gases below the spill edge (i.e. below the balcony), as:

$$\dot{m}_s = 0.025 \left(\dot{Q}_t W^2 \right)^{1/3} \tag{5}$$

By analogy, Law proposed a relationship for the mass flow rate of gases in the plume with respect to height above a virtual line source as:

$$\dot{m}_p \propto \left(\dot{Q}_t W_s^2 \right)^{1/3} \left(z_s + z_0 \right) \tag{6}$$

Law then utilised further experimental data from Morgan and Marshall [14] to plot $\dot{m}_p / \left(\dot{Q}_t W_s^2 \right)^{1/3}$ versus z_s. Law also used equation (5) to determine the value of $\dot{m}_p$ when $z_s = 0$ (which was included with the experimental data in the correlation). From this analysis, Law developed a simple formula to determine $\dot{m}_{p,3D}$ for a 3-D balcony spill plume (i.e. including end entrainment) channelled by screens below the balcony, given as:

$$\dot{m}_{p,3D} = 0.34\left(\dot{Q}_t W_s^2\right)^{1/3}\left(z_s + 0.075\right) \quad (7)$$

Law commented that the intercept value of 0.075 was equivalent to $0.15h_b$ which suggested that the virtual source of the plume was only a small distance below the spill edge. Equation (7) was therefore generalised to give:

$$\dot{m}_{p,3D} = 0.34\left(\dot{Q}_t W_s^2\right)^{1/3}\left(z_s + 0.15h_b\right) \quad (8)$$

Law proposed that equation (8) could be used as an alternative to the more complex BRE spill plume analytical method [7] for interim use until further analysis had been carried out.

Morgan [15] questioned the use of $q_{max,s}$ in the development of equation (8) due to its dependence on the flow behaviour upstream of the measuring region. He commented that this result may not necessarily apply to other geometries. Morgan also stated that the use of equation (5) to determine $\dot{m}_p$ when $z_s = 0$ appeared to be unjustified, and that this particular data point was given much greater weight than the experimental data. Morgan proposed a modified form of the Law correlation by only using the experimental data points and applying the effective layer depth correction to adjust the height of rise to get:

$$\frac{\dot{m}_{p,3D}}{\left(\dot{Q}_t W_s^2\right)^{1/3}} = 0.40\left(h_a - 1.26d_l\right) + 0.061 \quad (9)$$

Morgan stated that equation (9) implies that the virtual line source appears to be at a distance $0.3h_b$ below the spill edge. He suggested that equation (9) could possibly be used for design purposes, however, due to the inconsistencies in the location of the virtual line source, the BRE spill plume method was recommended if greater accuracy was required.

In response to these comments by Morgan, Law [16] agreed that the correlation given by equation (8) need not necessarily pass through the data point generated from equation (5). However, as the correlation was developed using an empirical approach, Law stated that equation (8) could reasonably be used for design purposes. Law also questioned whether the BRE spill plume method gave greater accuracy due to reservations on the application of the effective layer depth correction.

2.3 Thomas

Thomas [17] used the experimental data from Morgan and Marshall [13,14] and a relationship by Lee and Emmons [10] describing temperature rise in a line plume with respect to height, to determine the location of the virtual source of a spill plume. This relationship reduced to a form which was virtually identical to that of Yokoi, as used by Law. Analysis of the data showed that the location of the virtual source varied between $0.32h_b$ and $0.66h_b$ depending on the region where the gas temperature was measured (i.e. the approach flow or the plume). Thomas also noted that the location of the virtual source was dependent on the experimental geometry.

Thomas developed a simple spill plume formula to determine $\dot{m}_{p,3D}$ by utilising the Lee and Emmons relationship given by equation (2) and developing an explicit yet approximate relationship for the entrainment into the free ends of the plume by matching entrainment between a line plume and an axisymmetric plume at a large height of rise. This term was included into equation (2) to give a simple formula for a 3-D balcony spill plume channelled by screens below the balcony given by:

$$\dot{m}_{p,3D} = 0.58\left(\frac{\rho_{amb}^2 g}{c_{p,air} T_{amb}}\right)^{1/3} \dot{Q}_c^{1/3} W_s^{2/3}\left(z_s + z_0\right)\left(1 + \frac{0.22\left(z_s + 2z_0\right)}{W_s}\right)^{2/3} \quad (10)$$

Using typical values of ρ_{amb}, g, $c_{p,air}$ and T_{amb} this equation reduces to:

$$\dot{m}_{p,3D} = 0.21\dot{Q}_c^{1/3}\left(z_s + z_0\right)\left(W_s + 0.22\left(z_s + 2z_0\right)\right)^{2/3} \quad (11)$$

One of the uncertainties in equation (11) is the difficulty in selecting an appropriate location for the virtual source and Thomas [17] provides a number of alternative locations. Morgan et al. [7] suggest that the location of the virtual line source, as explicitly defined by Poreh et al. [3] [and later described in equation (19)] could be used as a reasonable approximation. Morgan et al. [7] also state that the term which describes entrainment into the free ends of the plume in equation (11) is a speculative correction and should be treated with caution.

2.4 Law (1995)

Law [18] used a similar analysis to that carried out in 1986 using further experimental data from Hansell et al. [19]. This analysis led to a revised formula which applies for a 3-D balcony spill plume channelled by screens below the balcony such that:

$$\dot{m}_{p,3D} = 0.31\left(\dot{Q}_t W_s^{\,2}\right)^{1/3}\left(z_s + 0.25h_b\right) \tag{12}$$

Law also noted that if no channelling screens were present beneath the balcony, the layer flow spread laterally and became diffuse and ill defined. Due to this lateral spread there was additional entrainment into the rising plume compared to a plume channelled by screens. To take this additional entrainment into account, Law used limited data from Hansell et al. [19] to develop an effective width of the unchannelled flow at the spill edge as

$$W_{e,s} = W_o + b \tag{13}$$

This effective width can be used in equation (12) to predict $\dot{m}_{p,3D}$ for unchannelled flow. Law states that equation (13) does not represent the actual width of the flow below the spill edge, but it is an effective property which should be specifically used to predict entrainment for unchannelled flow. Therefore, for unchannelled flow, the prediction of $\dot{m}_{p,3D}$ which is generally recognised to be an approximate solution to a complex smoke flow is given by:

$$\dot{m}_{p,3D,unchan} = 0.31\left(\dot{Q}_t\left(W_o + b\right)^2\right)^{1/3}\left(z_s + 0.25h_b\right) \tag{14}$$

2.5 CIBSE and BS 7974

A modified version of equation (12) is included within guidance given by CIBSE [2] and BS 7974 [20] but with the use of $\dot{Q}_c$ instead of $\dot{Q}_t$ to give

$$\dot{m}_{p,3D} = 0.36\left(\dot{Q}_c W_s^{\,2}\right)^{1/3}\left(z_s + 0.25h_b\right) \tag{15}$$

within the limit given by $z_s/W_s < 5$.

Equation (15) can also be applied for unchannelled flow using the effective width of plume given by equation (13). BS 7974 states that equation (15) is based on relatively few data and is intended for the assessment of smoke mass flow rates only.

2.6 Poreh, Morgan, Marshall and Harrison

Poreh et al. [3] used dimensional analysis to deduce a relationship between $\dot{m}_p$ and $\dot{Q}_c$ for a line plume. This assumed that the volumetric flux of ambient air into a unit length of the plume in the far field is a function of the buoyant flux per unit length and the distance from the virtual line source. A relationship to determine $\dot{m}_p$ for a spill plume was developed as:

$$\dot{m}_p = C\dot{Q}_c^{1/3}W_s^{2/3}\left(z_s + z_0\right) \tag{16}$$

where,

$$C = 0.3C_m\rho_{amb} \tag{17}$$

which is virtually identical to the Lee and Emmons line plume relationship given by equation (3).

Poreh et al. derived an explicit expression to determine the location of the virtual line source of the spill plume by examining the scenario when the height of rise of layer in the smoke reservoir was the same as the layer base at the spill edge (i.e. $z_s = -d_s$). In this case there is no additional entrainment into the smoke flow beyond the spill edge (i.e. $\dot{m}_p = \dot{m}_s$). Therefore, Poreh et al. deduced from equation (16) that,

$$\dot{m}_s = C\dot{Q}_c^{1/3}W_s^{2/3}\left(-d_s + z_0\right) \tag{18}$$

and hence,

$$z_0 = d_s + \frac{\dot{m}_s}{C\dot{Q}_c^{1/3}W_s^{2/3}} \tag{19}$$

By substituting equation (19) into equation (16), Poreh et al. derived a simplified formula to determine the mass flow rate of gases due to a spill plume which states:

$$\dot{m}_p = C\dot{Q}_c^{1/3}W_s^{2/3}\left(z_s + d_s + \frac{\dot{m}_s}{C\dot{Q}_c^{1/3}W_s^{2/3}}\right) \tag{20}$$

To determine the value of C, Poreh et al. correlated experimental data produced from four separate experimental studies of 2-D spill plumes described by Marshall and Harrison [21]. The correlation gave rise to $C = 0.16$, from which $C_m = 0.44$ using equation (17). Poreh et al. deduced that for $C_m = 0.44$, the entrainment coefficient $\alpha = 0.11$, which was lower than the value of $\alpha = 0.16$ proposed by Lee and Emmons [10].

It is convenient to rearrange equation (20) to express the amount of air entrained into the plume beyond the spill edge (i.e. $\dot{m}_p - \dot{m}_s$), which is given by:

$$\left(\dot{m}_p - \dot{m}_s\right) = C\dot{Q}_c^{1/3}W_s^{2/3}\left(z_s + d_s\right) \tag{21}$$

When expressed in this form, this method deals with the entrainment into the rotation region (i.e. $\dot{m}_p - \dot{m}_s$ at $z_s = 0$) by assuming it is the same as the entrainment into the virtual region of the plume, with the origin located at the base of the layer below the spill edge (i.e. $z_0 = d_s$). The Poreh et al. simplified formula to predict $\dot{m}_{p,2D}$ specifically for a 2-D balcony spill plume channelled by screens below the balcony is given as:

$$\left(\dot{m}_{p,2D} - \dot{m}_s\right) = 0.16\dot{Q}_c^{1/3}W_s^{2/3}\left(z_s + d_s\right) \qquad (22)$$

This method is given within guidance on balcony spill plumes in BS 7974 [20] but as it applies to 2-D balcony spill plumes, Morgan et al. [7] comment that this is likely to severely limit the range of scenarios in which this method can be applied.

2.7 Thomas, Morgan and Marshall

Thomas et al. [5] used a rigorous dimensional analysis in the development of a simplified spill plume formula. This method does not require an explicit term to specify the location of the virtual source, nor does it make the assumption of self-similar flow profiles in terms of temperature and velocity throughout the plume or a constant entrainment coefficient. The dimensional analysis produced a general formula based on dimensionless variables as:

$$\frac{\dot{m}'_p}{\dot{Q}'_c} = g\,\frac{z_s}{\dot{Q}'^{2/3}_c} + d\,\frac{\dot{m}'_s}{\dot{Q}'_c} + e \qquad (23)$$

where g , d and e are regression coefficients and

$$\dot{m}'_p = \frac{\dot{m}_p}{W_s}; \quad \dot{m}'_b = \frac{\dot{m}_s}{W_s}; \quad \dot{Q}'_c = \frac{\dot{Q}_c}{W_s} \qquad (24)$$

Thomas et al. statistically analysed the experimental data given by Marshall and Harrison [21] as used by Poreh et al. [3] to develop a formula for the 2-D balcony spill plume as:

$$\frac{\dot{m}'_{p,2D}}{\dot{Q}'_c} = 0.16\frac{z_s}{\dot{Q}'^{2/3}_c} + 1.2\frac{\dot{m}'_s}{\dot{Q}'_c} + 0.0027$$

$$\Rightarrow \dot{m}'_{p,2D} = 0.16\dot{Q}_c^{1/3}W_s^{2/3}z_s + 1.2\dot{m}_s + 0.0027\dot{Q}_c \qquad (25)$$

Thomas et al. also provided an alternative version of the spill plume formula provided by Poreh et al. [3] so that it was in the same form as equation (25). To achieve this, an empirical relationship to describe d_s was derived, using the data given by Poreh et al. and the method by Morgan [22] to predict the mass flow rate of gases from a fire compartment opening such that:

$$\frac{d_s\left(\dfrac{\dot{Q}_c}{W_s}\right)}{\left(\dfrac{\dot{m}_s}{W_s}\right)} = 2.5\left(1 + \frac{\dot{Q}_c}{c_{p,air}T_{amb}\dot{m}_s}\right) \qquad (26)$$

Thomas then substituted equation (26) into equation (22) to remove an element of redundancy requiring the calculation of both $\dot{m}_s$ and d_s. This then gave:

$$\frac{\dot{m}'_{p,2D}}{\dot{Q}'_c} = 0.16\frac{z_s}{\dot{Q}'^{2/3}_c} + 1.4\frac{\dot{m}'_s}{\dot{Q}'_c} + 0.0014 \qquad (27)$$

$$\Rightarrow \dot{m}'_{p,2D} = 0.16\dot{Q}_c^{1/3}W_s^{2/3}z_s + 1.4\dot{m}_s + 0.0014\dot{Q}_c$$

which Thomas states as being an acceptable alternative to equation (22). This form of the Thomas method is given within the guidance for 2-D balcony spill plumes in BS 7974 [20].

Thomas et al. also analysed the experimental data from Hansell et al. [19] for the 3-D balcony spill plume to give,

$$\frac{\dot{m}'_{p,3D}}{\dot{Q}'_c} = 0.34\frac{z_s}{\dot{Q}'^{2/3}_c} + 2.64\frac{\dot{m}'_s}{\dot{Q}'_c} - 0.0083$$

$$\Rightarrow \dot{m}'_{p,3D} = 0.34\dot{Q}_c^{1/3}W_s^{2/3}z_s + 2.64\dot{m}_s - 0.0083\dot{Q}_c \qquad (28)$$

with the limit $z_s/W_s < 3$.

Equation (28) has an entrainment coefficient (i.e. the value of g) of 0.34 which is similar to that of Law [i.e. the 0.31 in equation (12)] and CIBSE [i.e. the 0.36 in equation (15)] when correlating the same experimental data by Hansell et al.

Thomas also proposed an alternative expression to describe the total amount of entrainment into the free ends of a 3-D balcony spill plume using the data from Hansell et al. given by:

$$\dot{m}_{ends} = 0.09z_s\left(\frac{\dot{Q}_c}{W_s}\right)^{1/3} \qquad (29)$$

Thomas et al. state that equation (29) should be treated with caution as it applies when values of z_s/W_s are 'not too large' and the limit of this criterion remains unknown until further data is available.

2.8 Harrison and Spearpoint (2006)

Harrison and Spearpoint [6,23] carried out a series of experiments using a 1/10th physical scale model to provide new data describing entrainment into a 3-D balcony spill plume channelled by screens below the balcony. The experiments examined the scenario of both a flat ceiling and a downstand at the spill edge. Using the method by Poreh et al. [3]

a simplified formula was determined to predict $\dot{m}_{p,3D}$ as:

$$\dot{m}_{p,3D} = 0.20\dot{Q}_c^{1/3} W_s^{2/3} \left(z_s + d_s \right) + \dot{m}_s \qquad (30)$$

Harrison and Spearpoint [24] statistically analysed the data using the dimensional analysis by Thomas et al. [equation (23)] which gives an alternative design formula given by

$$\frac{\dot{m}'_{p,3D}}{\dot{Q}'_c} = 0.22 \frac{z_s}{\dot{Q}'^{2/3}_c} + 1.92 \frac{\dot{m}'_s}{\dot{Q}'_c} - 0.0042$$

$$\Rightarrow \dot{m}'_{p,3D} = 0.22\dot{Q}_c^{1/3} W_s^{2/3} z_s + 1.92\dot{m}_s - 0.0042\dot{Q}_c \qquad (31)$$

These formulae apply to a flow with either a flat ceiling or a downstand at the spill edge and should be applied within the limit given by $z_s / W_s < 2$. The values of the dominant entrainments coefficient from the Harrison and Spearpoint data (i.e. the 0.20 and 0.22) are significantly lower than the values obtained from correlating the Hansell et al. data (i.e. 0.31 to 0.36) for the 3-D balcony spill plume. This has given rise to further uncertainty in the prediction of entrainment for this scenario.

2.9 Valkist

Valkist [25] used numerical modelling to develop simple formulae for 3-D balcony spill plumes channelled by screens from wide compartment openings (i.e. greater than 7 m in width). These formulae were determined using data obtained from fourteen CFD simulations. Valkist used the predictions of $\dot{m}_{p,3D}$ to determine values of regression coefficients in the form given by:

$$\dot{m}_{p,3D} = B\dot{Q}_c^{1/3} W_s^{i} z_s + \dot{m}_{p,z_s=0} \qquad (32)$$

Equation (32) is of a similar form to the majority of spill plume models, but with the use of $\dot{m}_{p,z_s=0}$ (i.e. the value of $\dot{m}_p$ at z_s = 0) to describe the entrainment below the balcony without resorting to the assumption of a virtual line source. Correlating the data gave rise to:

$$\dot{m}_{p,3D} = 0.1936\dot{Q}_c^{1/3} W_s^{0.6174} z_s + \dot{m}_{p,z_s=0} \qquad (33)$$

which Valkist states as being a simple method to predict $\dot{m}_{p,3D}$.

The values of the regression coefficients are stated to an accuracy of four decimal places, as given by Valkist, although this is probably unjustified considering the uncertainty in the prediction of $\dot{m}_{p,3D}$ from fourteen simulations. Equation (33) gives a similar result to the formulae produced by Harrison and Spearpoint.

Valkist also provides a modified method, similar to equation (33), but with the use of an additional term to take into account the aspect ratio of the layer flow below the spill edge given by:

$$\dot{m}_{p,3D} = B\dot{Q}_c^{1/3} W_s^{i} \left(\frac{d_s}{W_s} \right)^{j} z_s + \dot{m}_{p,z_s=0} \qquad (34)$$

Correlating the data according to equation (34) gave rise to the values of regression coefficients given as:

$$\dot{m}_{p,3D} = 0.1060\dot{Q}_c^{1/3} W_s^{1.2523} \left(\frac{d_s}{W_s} \right)^{0.4151} z_s + \dot{m}_{p,z_s=0} \qquad (35)$$

Valkist states that the modified method given by equation (35) should be used in preference to the simple method given by equation (33).

The above equations require the user to determine $\dot{m}_{p,z_s=0}$. Valkist provides a general expression to determine this given by:

$$\dot{m}_{p,z_s=0} = B\dot{Q}_c^{1/3} W_s^{i} d_s^{j} \dot{m}_s^{n} \qquad (36)$$

Correlating the data according to equation (36) gave rise to the values of regression coefficients given as:

$$\dot{m}_{p,z_s=0} = 2.2628\dot{Q}_c^{1/3} W_s^{0.4923} d_s^{0.2426} \dot{m}_s^{0.5780} \qquad (37)$$

2.10 NFPA 92B

Guidance on balcony spill plume entrainment in NFPA 92B [2] has recently been updated following full scale experiments and numerical modelling carried out by Lougheed et al. [26] of the National Research Council (NRC), Canada. The guidance is dependent upon z_s and W_s and is as follows:

- For a 3-D balcony spill plume with z_s < 15 m,

$$\dot{m}_{p,3D} = 0.36\dot{Q}_t^{1/3} W_s^{2/3} \left(z_s + 0.25 h_b \right) \qquad (38)$$

- For a 3-D balcony spill plume where $z_s \geq$ 15 m and W_s < 10 m,

$$\dot{m}_{p,3D} = 0.59\dot{Q}_c^{1/3} W_s^{1/5} \left(z_s + 0.17 W_s^{7/15} h_b + 10.35 W_s^{7/15} - 15 \right) \qquad (39)$$

- For a balcony spill plume where $z_s \geq 15$ m and 10 m $\leq W_s \leq$ 14 m,

$$\dot{m}_{p,3D} = 0.2\dot{Q}_c^{1/3} W_s^{2/3} \left(z_s + 0.51 h_b + 15.75 \right) \quad (40)$$

Equation (38) is identical to the guidance in the 2005 version of NFPA 92B [27], with the exception that there is now a limit on its use (i.e. z_s < 15 m). Equation (38) is almost identical to the guidance given by CIBSE but with the use of $\dot{Q}_t$ in equation (38) instead of $\dot{Q}_c$ in equation (15). Equation (38) can be described in terms of $\dot{Q}_c$ and is given by [26]:

$$\dot{m}_{p,3D} = 0.41\dot{Q}_c^{1/3} W_s^{2/3} \left(z_s + 0.25 h_b \right) \quad (41)$$

NFPA 92B states that the NRC experiments show that equation (38) provides reasonable but conservative estimates for values of z_s below 15 m. This statement appears hard to justify as the ceiling height of the experimental atrium was 12.2 m and the height of the balcony was 5 m. Therefore, the height the atrium ceiling above the spill edge was approximately 7 m, with z_s generally ranging from 2 to 4 m in the experiments due to the limited smoke exhaust capacity of the experimental facility.

The NRC experimental results are anomalously high compared with measured entrainment from previous experimental studies and they do not appear to support the statement that equation (38) is conservative. This high entrainment is partially attributed to recirculation of the ceiling jet flow in the atrium space, as was also reported in one set of experiments by Marshall and Harrison [21] using a small collecting hood. However, the NRC results still produce significantly higher entrainment than the Marshall and Harrison 'small hood' experiments. Lougheed et al. [26] also attribute this additional entrainment to the trajectory of the plume in the near field, which is contrary to the findings reported by Harrison and Spearpoint [6] from small scale experiments. Lougheed et al. state that there are a number of factors that contributed to variation and uncertainty in the experimental results.

For z_s above 15 m, equations (39) and (40) predict a lower rate of entrainment with respect to height of rise compared to predictions using equation (38) for z_s below 15 m. The justification for a reduced rate of entrainment for z_s above 15 m is unclear, however, Lougheed [28] states that this height was chosen as a conservative estimate of the point of intersection between the simplified spill plume formula by Law [18] [equation (12)] and a line plume entrainment approximation. This analysis makes the assumption that the entrainment behaviour of the 3-D spill plume approaches that of a line plume at a height when the plume rises vertically into the atrium void, thus giving rise to a reduction in the rate of entrainment. However, this assumption neglects the effect of entrainment into the free ends of the plume on the overall entrainment behaviour and is not supported by the numerical modelling predictions by Lougheed et al. [26] which show that the rate of entrainment was constant (both below and above a height of 15 m) for z_s up to 50 m. Equations (39) and (40) were derived by adjusting the term describing the virtual source so that they matched the prediction of entrainment using equation (38) at a height of 15 m. It is interesting to note that the location of the virtual source is dependent on h_b in equations (38) and (40), yet it is dependent on both h_b and W_s in equation (39). It is unclear why there is inconsistency in the terms to describe the location of the virtual source between these equations.

NFPA 92B states that a design based on equations (39) and (40) (i.e. z_s > 15 m) should be compared with predictions for an axisymmetric plume scenario and the higher mass flow rate used for the design of the atrium smoke management system. It is also recommended that this scenario is supported by a CFD modelling study.

NFPA 92B states that the NRC experiments demonstrate that the above equations can be applied for unchannelled flows for compartment opening widths between 5 m and 14 m using the effective lateral extent of the flow given by equation (13). This statement should be treated with caution as Ko et al. [29] report that mass loss was observed from the ends of the balcony in the NRC experiments, therefore it was not possible to examine the true lateral extent of the unchannelled flow and plume.

2.11 Harrison and Spearpoint (2008)

Harrison and Spearpoint [24] carried out a series of experiments using a 1/10th physical scale model to provide new data to systematically characterise entrainment into a balcony spill plume generated from a flow channelled by screens beneath the balcony. They demonstrated that existing simplified design formulae for the 2-D balcony spill plume apply generally for plumes generated from a range of fire compartment geometries and proposed a further simplified formula for the 2-D plume given by:

$$\dot{m}_{p,2D} = 0.16\dot{Q}_c^{1/3} W_s^{2/3} z_s + 1.34\dot{m}_s \quad (42)$$

Harrison and Spearpoint state that the rate of entrainment into a 3-D balcony spill plume appears to be specifically dependent on the characteristics of the layer flow below the balcony edge. 3-D plumes generated from narrow openings entrain air at a greater rate with respect to height compared to those generated from wider openings. The rate of entrainment appears to be dependent on the contribution of the end entrainment in the overall entrainment process. They propose a simplified design formula for the 3-D plume by developing a general empirical expression to explicitly describe the entrainment of air into the ends of the plume which gives:

$$\dot{m}_{p,3D} = 0.16\dot{Q}_c^{1/3}\left(W_s^{2/3} + 1.56d_b^{2/3}\right)z_s + 1.34\dot{m}_s \quad (43)$$

Harrison and Spearpoint state that this formula can be applied more generally compared to currently available formulae and that this work goes some way to explain and reconcile differences in entrainment reported between previous studies.

3. SIMPLIFIED DESIGN FORMULAE FOR THE ADHERED SPILL PLUME

Fig. 2 shows a schematic drawing of an adhered spill plume. In this case, there is no balcony or horizontal projection beyond the compartment opening, and a wall projects vertically above the top of the opening. The smoke layer below the compartment opening will then rotate at the spill edge (i.e. the compartment opening in this case). The subsequent plume will then adhere to the vertical wall above the opening as it rises. This type of plume is also known as a single-sided spill plume.

There is limited experimental data on entrainment into adhered plumes with a single simplified formula developed by Poreh et al. [30] for the 2-D adhered plume. There are no robust simplified formulae to predict entrainment for the 3-D adhered plume, however, CIBSE [2] give a formula to provide an approximate solution for this scenario. These formulae are described below.

3.1 Poreh, Marshall and Regev

Poreh et al. [30] give a simplified formula for a 2-D adhered spill plume based on limited experimental data obtained by Marshall [31]. Poreh et al. correlated the data using the method described in section 2.6 to determine C = 0.075 and thus:

$$\dot{m}_{p,2D} = 0.075\dot{Q}_c^{1/3}W_s^{2/3}\left(z_s + d_s\right) + \dot{m}_s \quad (44)$$

It is interesting to note that the value of C is approximately half that of a 2-D balcony spill plume (i.e. 0.16). This is not surprising considering that entrainment only occurs into one side of an adhered plume, compared to two sides in a balcony spill plume.

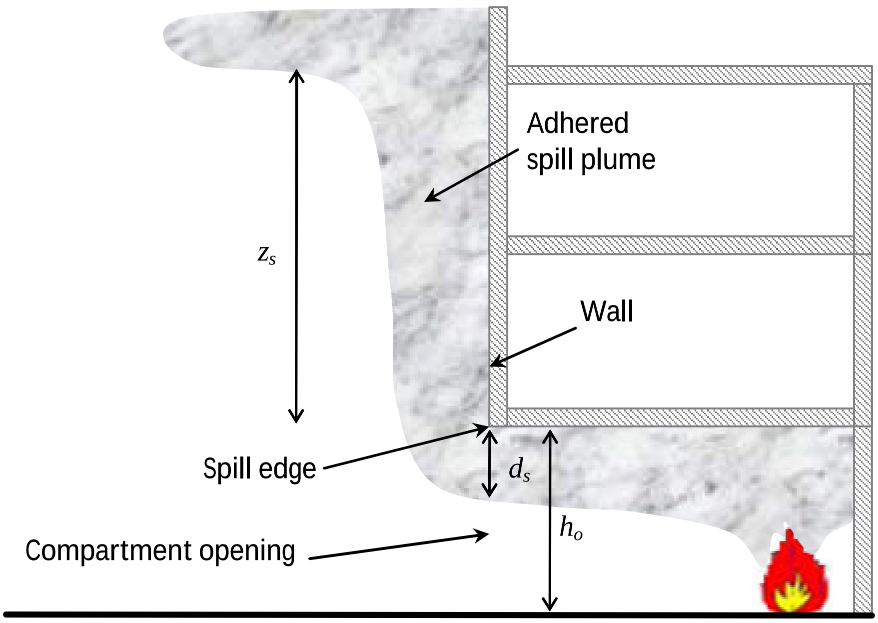


Fig. 2: An adhered spill plume

3.2 CIBSE

CIBSE [2] provides a formula to determine the mass flow rate of a spill plume above a compartment opening without a balcony as:

$$\dot{m}_{p,3D} = 0.23\dot{Q}_c^{1/3}W_s^{2/3}\left(z_s + h_o\right) \quad (45)$$

CIBSE state that this equation can be used whether or not there is a wall above the opening, so therefore can apply to the adhered plume scenario. Equation (45) was derived using the adhered plume experimental data from Hansell et al. [19] which was subsequently increased by 50%, and data from Porter [32] from post-flashover fires. It is unclear why the Hansell et al. data was increased by 50% (presumably to be conservative), nor is the derivation of this formula published elsewhere.

4. CONCLUDING REMARKS

This paper describes the basis and derivation of simplified entrainment calculation methods for the thermal spill plume and provides a useful source of reference for Fire Engineers in smoke management design.

These simplified formulae have the general form $\dot{m}_p = a\dot{Q}_c^{1/3} f\left(W_s, z_s\right) + \dot{m}_{p,z_s=0}$ where the entrainment coefficient a describes the rate of entrainment above the spill edge and is dependent on the nature and type of spill plume. There has been much debate regarding an appropriate value of a for design purposes due to differences in measured entrainment between previous experimental studies. Care should be taken when using these formulae to correctly specify whether $\dot{Q}_t$ or $\dot{Q}_c$ should be used in the calculation.

An assessment of the performance and appropriateness of these formulae is outside the scope of this paper, however an assessment of selected formulae has been made by Harrison [33] by comparison with new experimental data. Harrison also provides new simplified design formulae which apply more generally for a range of geometries and plume types compared to existing methods and reconciles differences in measured entrainment (i.e. the value of a) between previous studies.

NOMENCLATURE

a — entrainment coefficient, $kgms^{-1}kW^{-1/3}$
B — entrainment coefficient, $kgms^{-1}kW^{-1/3}$
b — balcony breadth, m
C — entrainment coefficient, $kgms^{-1}kW^{-1/3}$
C_m — dimensionless entrainment coefficient
c_p — specific heat, $Jkg^{-1}K^{-1}$
d — depth of gas layer, m
g — acceleration due to gravity, ms^{-2}
h — height above floor, m
$\dot{m}$ — mass flow rate of gases, kgs^{-1}
$\dot{m}_{ends}$ — mass flow rate entrained into the free ends of the plume, kgs^{-1}
$\dot{Q}_c$ — convective heat flow in the gas layer below the spill edge, kW
$\dot{Q}_t$ — total heat generated by the fire, kW
T — absolute gas temperature, K
W — width or lateral extent, m
z_s — height of rise of plume above the spill edge, m
z_0 — height of virtual line source below the spill edge, m

Greek Symbols

a — entrainment constant for plume
g — regression coefficient
d — regression coefficient
e — regression coefficient
q — temperature above ambient (°C)
r — density (kgm^{-3})
l — an empirical thermal plume constant

Subscripts

amb — an ambient property
a — variable evaluated in an atrium
air — a property of air
b — a property of the balcony
e — an effective property
l — a property of a smoke layer
max — a maximum value
o — a property of the fire compartment opening
p — variable evaluated in the plume at an arbitrary height of rise
s — variable evaluated in the gas layer flow below the spill edge
unchan — a property of a balcony spill plume without channelling screens below the balcony
2D — two-dimensional spill plume
3D — three-dimensional spill plume

Superscripts

i — regression coefficient
j — regression coefficient
n — regression coefficient

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