TEMPERATURE DISTRIBUTION SCHEDULE OF A BODY AT A COMPLEX THERMAL EFFECT FOR A FIRE HAZARD ANALYSIS

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ABSTRACT

At a complex thermal impact (extreme temperature of milieu and internal heat generation sources), the analytical solution to transient conduction problem for the homogeneous body with protuberant geometry and boundary conditions of the third kind has been obtained. Since the superposition principle commonly used for such solutions is not applicable, the problem has been broken into two parts, each of which deals with only one thermal impact. Both internal and external factors determine the temperature increase rate of the body, but their contribution is quite different with respect to thermal and physical properties of the body. For two solid materials with different properties, the temperature increase rates (due to internal heat generation sources) are comparable, while such rates (due to external temperature impact) are rather different. The body (equipment) with lower thermal conductivity would warm up lower than the equipment with higher thermal conductivity. Hence the equipment with low thermal conductivity (and low diffusivity) has higher opportunity to keep functioning long at the impact of an extreme milieu temperature and internal heat generation sources.

1. INTRODUCTION

Nowadays, a fire hazard analysis requires not only information about possible ways of a fire development, but assessment or prediction of functioning some particular equipment that is located within a dangerous zone of the fire. Such an assessment or prediction of the vital equipment functioning are impossible without understanding and quick estimating the temperature distribution processes within this equipment during a fire impact. Because the information about real fire parameters might be rather approximate, it is reasonable to assume that a temperature schedule of a fire that influences on equipment and warms it is a step-function. Such a problem for the homogenous body (equipment) with protuberant form has been solved in reference [1]. But there is quite often a case when the body (object) itself has its own sources of the heat generation (because, for example, the body (equipment) contains elements that convert electrical energy to the thermal one). Thus, the body is under effect not only the external factor (a fire), but also the internal one caused by the heat generation process within the body.

The main goal of this paper is the analytical solution to transient-heat-conduction problem for a body with protuberant form subject to an external step-function thermal assault and an internal heat generation effect. Such a solution can be used for quick estimation of the threat posed by an unexpected thermal event, or it can be applied for some sort of a risk assessment. The need to develop an analytical solution versus numerical one is relayed to some additional reasons: first, a real temperature variation of milieu during such an accidental phenomenon as a fire, a vapor leakage, etc. might be determined only roughly (because of complexity and low predictability of these phenomena). Consequently, it is not necessary to apply accurate numerical methods. Accuracy of any solution method of a physical problem should not and could not exceed the accuracy of the original formulation of this problem [2]. Second, the analytical nature of the solution not only can grasp the whole group of different cases, but also might provide some insight into important parameters of the accidental scenario. In other words, an analytical solution is more powerful than a numerical one with respect to contemplating and generalizing capabilities of any physical or natural difficult phenomenon.

Let us formulate the problem more precisely. At some particular moment of time $\tau = 0$, a body with protuberant form and homogeneous structure is affected by its own heat generation sources that are evenly located throughout this body. Such a uniform heat generation rate per unit volume can be described as:

$$\psi(\tau) = \begin{cases} 0, & \text{if} \quad \tau < 0 \\ \psi, & \text{if} \quad \tau \geq 0 \end{cases} \tag{1}$$

Heat-transfer and physical properties of the body, such as thermal conductivity, λ, constant-pressure specific heat, c, and density, γ, are known.

Temperature of milieu for this period is $t = 0$. The body temperature, T, begins to grow because of the internal heat generation sources and, in some period of time, it might be stabilized. Then, at some another particular moment, $\tau_* \geq 0$, a step-function type thermal assault begins to act (the milieu temperature sharply rises up to $t = t_*$), and the body intensively warms. This external thermal effect might be described as follows:

$$t(\tau) = \begin{cases} 0 & \text{if } \tau < \tau_* \\ t_* & \text{if } \tau \geq \tau_* \end{cases} \quad (2)$$

Since this moment τ_*, the milieu temperature t_* is larger (very often, much larger) than the body temperature, the heat transfer between the body and milieu happening by convection also rises up, and the average heat transfer coefficient h_* also becomes bigger than h before the moment $\tau = \tau_*$. Thus in general,

$$h_* > h$$

That can be explained by the average increase of the gas movement of milieu within an enclosure where a fire occurs and, hence, intensification of the convection processes around the body as a whole.

To solve the problem of determining the temperature distribution schedule within a body due to complex thermal effect, the Laplace transform method can be used. Dr. Yaryshev et al. [2] showed that for such a case, it is possible to design two different transfer functions of the body for each thermal effect separately. Specifically, for our problem, the transfer functions can be represented as ratios of two infinite polynomials [2]:

$$Y_e(\rho, s) = \frac{1 + \sum_{k=1}^{m} b_k s^k}{1 + \sum_{k=1}^{n} a_k s^k} \quad (3)$$

$$Y_i(\rho, s) = \frac{a}{L^2} \frac{\sum_{k=1}^{m} (c_k - d_k) s^{k-1}}{1 + \sum_{k=1}^{n} c_k s^k} \quad (4)$$

where a_k, b_k, c_k, d_k are coefficients which can be calculated as follows:

$$a_k = \frac{\Gamma(\nu+1)(1 + Bi/2k)}{2^{2k-1}\Gamma(k)\Gamma(k+\nu+1)Bi} \times (L^2/\alpha)^k \quad (5)$$

$$b_k = \frac{\Gamma(\nu+1)}{2^{2k} k\Gamma(k)\Gamma(k+\nu+1)} (L^2\rho^2/\alpha)^k \quad (6)$$

where Γ is gamma-function of a corresponding substantial argument. ν is a function of the shape-factor of the body N. It can be determined as [2]:

$$\nu = (N-1)/2 \quad (7)$$

A shape-factor of the body, N, depends on its external surface, S, its volume, V, and characteristic half-length, L, of this body. According to [2], it might be estimated as:

$$N = V/SL - 1 \quad (8)$$

For example, for a regular sphere, an infinite cylinder, and an infinite plate, a shape-factor, N, equals 2, 1, and 0 correspondingly.

Following [2], the temperature distribution schedule of the body, $T(\rho, \tau)$, is the output due to both external input, $t(\tau)$, and internal input, $\psi(\tau)$, described by equations (1) and (2). Thus, the Laplace transform of the temperature distribution schedule within the body, $\overline{T}(\rho, s)$, is the sum of two products:

$$\overline{T}(\rho, s) = \overline{\psi}(s) Y_i(\rho, s) + \bar{t}(s) Y_e(\rho, s) \quad (9)$$

where $\rho = r/L$ is the relative polar coordinate of the body; s is the Laplace complex variable of time; $\overline{\psi}(s)$ and $\bar{t}(s)$ are the Laplace transforms of the internal and external inputs correspondingly.

In reference [1], it was shown that transfer functions (see equations (3) and (4)) might be considerably simplified by keeping only the first three members of the polynomials. So, they can be represented as:

$$Y_e(\rho, s) = \frac{1 + b_1 s + b_2 s^2 + b_3 s^3}{1 + a_1 s + a_2 s^2 + a_3 s^3} \quad (10)$$

$$Y_i(\rho, s) = \frac{\alpha}{L^2} \frac{(c_1 - d_1) + (c_2 - d_2)s + (c_3 - d_3)s^2}{1 + c_1 s + c_2 s^2 + c_3 s^3} \quad (11)$$

From equation (9), it is possible to see that the temperature distribution schedule of the body can be determined as a sum of two different items that can be estimated separately. The first item (component) is the temperature distribution variation of the body due to internal heat generation sources. Let us look into this component, first. For period of time $0 < \tau < \tau_*$, it is necessary to find an inverse Laplace transform of the first component in equation (9) that, in fact, will be a solution of the problem:

$$T(\rho,\tau) = \frac{\psi}{c\gamma}(c_1 - d_1)[1 + A\exp K\tau + \exp U\tau(B\cos V\tau + C\sin V\tau)] \qquad (12)$$

where

$$A = \frac{1 + f_1K + f_2K^2}{c_3K[(K-U)^2 - V]} \qquad (13)$$

$$B = \frac{1}{c_3P}\{\frac{K-2U}{U^2+V^2} - f_1 - f_2K\} \qquad (14)$$

$$C = \frac{U^2 - UK - V^2}{c_3P(U^2+V^2)} + \frac{f_1(U-K) + f_2(U^2+V^2-UK)}{c_3VP} \qquad (15)$$

$$P = K^2 + U^2 + V^2 - 2UK \qquad (16)$$

$$f_1 = \frac{c_2 - d_2}{c_1 - d_1}, \qquad f_2 = \frac{c_3 - d_3}{c_1 - d_1} \qquad (17)$$

$K, U \pm iV$ are the roots of equation:

$$c_3s^3 + c_2s^2 + c_1s + 1 = 0 \qquad (18)$$

These roots can be determined by well-known formulas [e.g. 3].

Equation (12) together with equations (13) to (17) describe the temperature field variation of the body under internal thermal impact (which begins to act at moment $\tau = 0$). After some period of time (τ is still less than τ_*), the temperature distribution schedule of the body becomes steady or nearly steady. (Precisely, it happens when $\tau \to \infty$, hence $s \to 0$). Consequently for such a case, equation (8) can be converted into:

$$T(\rho,\tau) = \frac{\psi}{c\gamma}(c_1 - d_1) \qquad (19)$$

In practice, the case of the steady temperature distribution schedule of the body due to internal thermal effect is quite common. It is consistent with real situation that takes place, for example, within an electrical equipment that was turned on at some moment and its electrical/electronic elements warmed up and stabilized for a while.

During the second phase, when the external temperature effect begins to act ($\tau \geq \tau_*$), temperature of milieu becomes bigger than the body temperature, consequently, it is possible to assume that the heat energy release due to internal sources out from the body will be closed. Such a situation is equivalent to the one, when the heat transfer coefficient between the body and milieu equals 0. So, since moment $\tau \geq \tau_*$, in order to find the temperature distribution variation within the body due to internal thermal effect, it is necessary to determine the limit of equation (12) at the Biot number $Bi = \frac{h}{L\lambda} \to 0$. This limit is:

$$T(\rho,\tau) = \frac{\psi}{c\gamma}\tau \qquad (20)$$

Equation (20) is rather interesting. It shows that if the milieu temperature becomes bigger than the body temperature, then $Bi \to 0$, and the temperature schedule due to internal heat generation does not depend on a specific point within the body, and at each point of the body, the temperature rise rate has the same meaning. This temperature rate is directly proportional to the heat generation rate, ψ, and inversely proportional to the volumetric heating capacity of the body, $c\gamma$. Remarkably, the temperature rate does not depend at all on the thermal conductivity of the body, λ. Thus, for all period of time, the temperature schedule of the body due to internal thermal effect can be determined as:

$$T(\rho,\tau) = \begin{cases} T^0(\rho,\tau) & \text{if} \quad \tau < \tau_* \\ T^0(\rho,\tau) + \dfrac{\psi}{c\gamma}(\tau - \tau_*) & \text{if} \quad \tau \geq \tau_* \end{cases} \qquad (21)$$

where $T^0(\rho,\tau)$ is the temperature schedule, estimated by either equation (12) or (19).

Simplicity and clearness of equation (20) shows that parameter $\frac{\psi}{c\gamma}$ has its own value, and it can be used as some sort of criterion of vulnerability (survivability) for the vital equipment disturbed by an extreme temperature impact of milieu (including a fire). Numerically, parameter $\frac{\psi}{c\gamma}$ equals the rate of temperature increase (number of degrees per time unit) at any point of the body due to its own heat generation sources under external extreme temperature of milieu. The bigger parameter $\frac{\psi}{c\gamma}$, the faster temperature increase rate, hence the shorter timing of functioning the vital equipment disturbed by this extreme milieu temperature. As an example, meanings of thermodynamic properties and such criteria at the heat generation rate per volume unit ψ = 3500 Jm^{-3} for different materials and subjects are represented in Table 1. (Thermodynamic properties were used from reference [4]).

Data represented in Table 1 permits to make one more interesting conclusion. Even though thermodynamic properties of materials are significantly different from one another, the volumetric heating capacity, $c\gamma$, varies only a little. So, for the body made from such kind of materials, the temperature increase rate due to internal heat generation sources would mostly depend on the rate of this generation, ψ, rather than the type of the body material (its thermal and physical properties).

On the other hand, thermal conductivity, λ, and hence, thermal diffusivity, $\frac{\lambda}{c\gamma}$, greatly vary from one material to another. So, the temperature variation rate of a body due to an external effect would greatly depend on the type of the body material. The bigger thermal conductivity, the higher temperature increase rate of the body due to extreme temperature impact of milieu. Thus, under other even circumstances, the body (equipment) with lower average thermal conductivity would warm up lower than the equipment with higher thermal conductivity. Hence the equipment unit with low thermal conductivity has higher ability to keep functioning long; therefore, it has bigger survivability (smaller vulnerability) at the extreme temperature impact of milieu.

Let us discuss, now, the second part of our problem, in other words, the second component of equation (9). The temperature distribution schedule of the body due to external effect (formula 1), has been already founded in reference [1]. It is also necessary to emphasize that, for the external thermal effect, the heat-transfer coefficient is h_*, and the Biot number is not equal to 0. Following reference [1], the temperature variation within the body due to external thermal effect will be:

$$T(\rho,\tau) = t_*\{1 + M\exp K_*(\tau-\tau_*) + \exp U_*(\tau-\tau_*)[N\cos V_*(\tau-\tau_*) + R\sin V_*(\tau-\tau_*)]\} \quad (22)$$

where

$$M = \frac{1 + b_1K_* + b_2K_*^2 + b_3K_*^3}{a_3K_*[(K_* - U_*)^2 + V_*^2]} \quad (23)$$

$$N = \frac{2U_*K_* - K_*^2}{P_*} - \frac{b_1K_* + b_2K_*^2 + b_3(2U_*K_* - U_*^2 - V_*^2)}{a_3P_*} \quad (24)$$

Table 1

Material	$\gamma, \frac{kg}{m^3}$	$c, \frac{J}{kgK}$	$\lambda, \frac{W}{mK}$	$\frac{1}{c\gamma}\times 10^7, \frac{m^2K}{N}$	$\frac{\psi}{c\gamma}, \frac{K}{min}$
Aluminum (alloy)	2770	875	177	4.126	0.087
Copper (pure)	8933	385	401	2.908	0.061
Germanium	5360	322	59.9	5.79	0.122
Silicon carbide	3160	675	490	4.69	0.098
Steel (plane carbon)	7854	434	60.5	2.93	0.062
Rubber, vulcanized	1100	2010	0.13	4.52	0.095
Textolite	1350	1480	0.30	5.01	0.105
Gold	19300	129	317	4.02	0.084
Silver	10500	235	429	4.05	0.085
Titanium	4500	522	21.9	4.26	0.089
Uranium	19070	116	27.6	4.52	0.095
Quartz glass	2650	892	1.45	4.231	0.089

$$R = \frac{U_*^2 - U_*K_* - V_*^2}{a_3P_*(U_*^2 + V_*^2)} + \frac{b_1(U_* - K_*) + b_2(U_*^2 + V_*^2 - U_*V_*)}{a_3V_*P_*} +$$

$$\frac{b_3[U_*^2(U_* - K_*) + V_*^2(U_* + K_*)]}{a_3 V_* P_*} \quad (25)$$

$$P_* = K_*^2 + U_*^2 + V_*^2 - 2U_* K_* \quad (26)$$

where K_* and $U_* \pm V_*$ are the roots of equation:

$$1 + a_1 s + a_2 s^2 + a_3 s^3 = 0 \quad (27)$$

Taking into account all the above, the entire temperature distribution schedule within the body under both internal and external thermal effects can be estimated as:

$$T(\rho,\tau) = \begin{cases} T^0(\rho,\tau) & \text{if} \quad \tau < \tau_* \\ T^0(\rho,\tau_*) + \frac{\psi}{c\gamma} + T^*(\rho,\tau) & \text{if} \quad \tau \geq \tau_* \end{cases} \quad (28)$$

where $T^*(\rho,\tau)$ is determined by equation (22).

A methodical error of equation (28) and the proposed method as a whole can be estimated according to the next circumstances. First, some portion of the methodical error causes using approximate transfer functions of the body 5, 6 instead of "precise" transfer functions 3 and 4. This portion of the methodical error is tiny at any moment of time $\tau > \tau_*$. Rather, the further a current moment of time, τ, from point τ_*, the more and more tiny this portion of methodical error is. The second portion of methodical error is caused by irregularity of the temperature distribution within the body just before the moment τ_*. From equation (20), it is possible to see that if $Bi \to 0$, then the temperature rates are the same at any point of the body. So before moment τ_*, the temperature field of the body is distributed unevenly, then after this moment, this temperature distribution irregularity must disappear for a while. And as long as a current moment of time is far from moment τ_*, such irregularity must become less and less. On the other hand, equation (28) together with the others do not represent this phenomenon. Hence, it has a methodical error. The maximum value of this error can be estimated as a possible maximum difference of local temperatures within the body just before the moment τ_*. This maximum temperature difference takes place between two local points (surfaces) of the body. One of them is the center of the body ($\rho = 0$) and the second one is the outside surface of the body ($\rho = 1$). Thus, this maximum temperature difference can be estimated as:

$$\Delta T_{max} = T(0,\tau) - T(1,\tau) \quad (29)$$

It is reasonable to assume that the maximum temperature difference occurs, when the temperature schedule of the body due to internal thermal effect becomes steady (so, $\tau \to \infty$). Thus, equation (29) can be transformed to:

$$\Delta T_{max} = T(0,\infty) - T(1,\infty) \quad (30)$$

Taking into account equations (5) and (6), it is possible to get:

$$\Delta T_{max} = \frac{WL^2}{4(1+\nu)\lambda} \quad (31)$$

The maximum relative methodical error can be estimated as a ratio between ΔT_{max} and temperature of the external assault t_*:

$$\Delta = \frac{WL^2}{4(1+\nu)\lambda} \times \frac{1}{t_*} \quad (32)$$

For example, if the body that has spherical form (radius r = 1 m; $\lambda = 0.2\ Wm^{-1}K^{-1}$) is influenced by internal thermal effect ($\psi = 3500\ Wm^{-3}$) and external temperature assault ($t_* = 600^o C$), then the maximum relative error is not more than 5% from the value of the external temperature t_*. (In that example, all represented numbers are common for electronic/electrical apparatuses that have free convection cooling system and affected by the high temperature of the fire).

2. SUMMARY

- At a complex thermal impact (extreme temperature of milieu and internal heat generation sources), the analytical solution to transient conduction problem for the homogeneous body with protuberant geometry and boundary conditions of the third kind has been obtained. Such a solution has been received by the use of the Laplace transform method and design of two transfer functions of the body with respect to each thermal impact separately. Corresponding formulas for calculating temperature distribution schedule of the body have been developed.

- A manner and formula for estimating methodical error of the obtained solution has been proposed. This error is mostly caused by irregularity of the temperature distribution within the body just before the moment when an extreme temperature of milieu begins to act. Its value is directly proportional to the heat generation rate per unit volume, the characteristic half-length of the body in square power, and inversely proportional to the thermal conductivity of the body (see equations (31) and (32)).

- Since affecting the extreme temperature of milieu, the boundary conditions for the internal thermal impact convert into the case when $Bi \to 0$. In other words, the energy of heat generation sources is "locked" within the body. In fact, superposition principle commonly used for such problems is not applicable. Therefore, the problem of estimating temperature distribution schedule due to heat generation impact has been broken into two parts, each of which has its own boundary conditions. The first conditions ($Bi = \frac{h}{\lambda L}$) are applicable for the period of time $\tau < \tau_*$, while the second ones ($Bi \to 0$) are appropriate if $\tau \geq \tau_*$.

- When an extreme temperature of milieu is getting bigger than the body temperature, the temperature increase rate of the body due to heat generation sources can be described as:

$$T(\rho, \tau) = \frac{\psi}{c\gamma}\tau$$

- The equation shows the temperature rate is the same for each point (place) of the body, and it is directly proportional to the heat generation rate, ψ, and inversely proportional to the volumetric heating capacity of the body, $c\gamma$. Because, the last property varies only a little from one solid material to another, the temperature rate, in fact, mostly depends on the rate of heat generation, ψ, rather than the type of the body material (its thermal and physical properties). Such a conclusion is getting much true if the body is composed from different materials.

- On the other hand, thermal conductivity, λ, and hence, thermal diffusivity, $\frac{\lambda}{c\gamma}$, can considerably vary from one material to another. So, the temperature variation rate of a body due to an external temperature effect greatly depends on the type of the body material. The bigger thermal conductivity and thermal diffusivity, the higher temperature increase rate of the body due extreme temperature impact of milieu.

- Though the temperature distribution schedule of the body is determined by both internal and external factors, their influence is quite different with respect to thermal and physical properties of the body. For two materials with quite different properties, the temperature increase rates due to internal heat generation sources are comparable, while such rates due to external temperature impact are rather different. The body (equipment) with lower average thermal conductivity would warm up lower than the equipment with higher thermal conductivity. Hence the equipment with low thermal conductivity (and low diffusivity) has higher opportunity to keep functioning long; therefore, it has bigger survivability (or smaller vulnerability).

NOMENCLATURE

T	temperature of body, K
$\check{T}$	Laplace transform of temperature function
τ	time, s
s	Laplace complex variable of time, s^{-1}
λ	thermal conductivity of body, $Wm^{-1} K^{-1}$
γ	density of body, kgm^{-3}
c	constant-pressure specific heat, $Jm^{-3}K^{-1}$
h, h_*	heat-transfer coefficients, $Wm^{-2}K^{-1}$
t_*	temperature of external thermal effect, K
ψ	heat generation rate per unit volume of the body, Wm^{-3}
Y_i, Y_e	Laplace transfer functions of the body for internal and external thermal effects
S	external surface area of the body, m^2
V	volume of the body, m^3
L	characteristic half-length (radius) of the body, m
r	polar coordinate of the body, m
$\rho = r/L$	relative polar coordinate of the body, -
Γ	gamma-function of a corresponding substantial argument, -
ν	function of the shape-factor of the body N, -
N	shape factor of the body, -
a_k, b_k, c_k, d_k	coefficients of equation (3) and (4)
A, B, C, D	parameters of equations (13) to (17)
K, K_*	real roots of equations (18) and (27)
$U \pm iV, U_* \pm iV_*$	complex roots of equation (18) and (27)
$\alpha = \frac{\lambda}{c\gamma}$	thermal diffusivity of the body, m^2s^{-1}
$Bi = \frac{hL}{\lambda}$	Biot number, -

k number of coefficients in equation (3) and (4)

Δ maximum relative methodical error (equation (32))

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