A STUDY ON PLUME EQUATION IN A CHAMBER WITH FORCED VENTILATION

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ABSTRACT

Thermally-induced plume in a chamber under forced ventilation will be studied in this paper. Plume equations derived under natural ventilation available in the literature will be reviewed first. Computational Fluid Dynamics are then applied to study two ‘models’ induced by a heat source in a chamber with forced ventilation. They are the parallel flow model and the jet flow model. Based on the plume expression in chambers with natural ventilation, two plume equations under forced ventilation are then derived from seven sets of CFD simulations. There are four cases on different thermal power of the point heat source; and three cases on plume in chamber with forced ventilation under three different air speeds. Symbolic mathematics are used for solving some equations concerned.

1. INTRODUCTION

Plume induced by a thermal source such as a fire in a chamber under forced ventilation condition is very complicated. Most of the plume equations in the literature [e.g. 1-11] were derived under natural ventilation condition with free boundary conditions. However, the equations concerned might not be applicable in a chamber with forced ventilation. The following questions are raised:

- Can those equations be applied under forced ventilation conditions?
- How is the situation under forced ventilation conditions?

To answer the above questions, equations under forced ventilation condition in a chamber will be studied in this paper. The plume equation under forced ventilation can be described by two flow models as in Fig. 1:

- A parallel flow model
  The fluid is assumed to be incompressible flow through two horizontal parallel planes with constant pressure difference. The direction of flow is taken along the horizontal axis, with velocities in other directions taken as zero. Velocity distribution is laminar and plane parallel [11], could be regarded as a forced ventilation condition [12] in a chamber.

- A jet flow model
  Incompressible fluid flow is assumed through a fine nozzle, and then diffused in a free boundary space [13]. The resultant plume is then induced by as a point heat source on the floor.

These two models will be studied by Computational Fluid Dynamics. Associated plume equations are then discussed in this paper.

2. JET FLOW MODEL

A chamber of length 4 m, width 4 m, and height 3 m as shown in Fig. 2 was considered. A point heat source with radius 0.1 m and height 0.1 m was located at the center of the chamber as shown in the figure. There are no walls in the perimeter, just a floor and a ceiling. This was labeled as the ‘fire case with an open lid’ by Galea and coworkers [12].

Air flow induced by the thermal source was simulated with the following assumptions [1]:

- The rate of entrainment at the plume edge is proportional to some characteristic velocity at that height.
- The profiles of mean vertical velocity and mean buoyancy force in horizontal sections are of similar form at all heights.
- The largest local variations of density in the field of motion are small in comparison with some chosen reference of density, this
reference being taken as the density of the ambient fluid at the level of the source.

- The scale of the initial laminar zone is small compared with the subsequent zone of turbulent convection.
- Notation for either two-dimensional flow over a line source or axially symmetric flow over a point source may be reduced to that shown in Fig. 3.
- Pressure is distributed uniformly throughout the field of motion.
- Transverse forces are ignored in comparison with those in the vertical direction.
- Turbulent mixing in the vertical direction is ignored in comparison with that in the horizontal.
- No mixing between the plume and ambient air flow.

- Floor, ceiling and walls of the chamber are made of adiabatic materials.
- No viscosity between horizontal sections.
- The form of the plume border is like a cone, which can be considered as a source point extended to a circle with the rise of the plume.

A typical plume model is shown in Fig. 3. Under the above assumptions and taking the point heat source on the floor, four cases with different thermal pressure were simulated for studying the plume equation:

- Case 1: 1 kW
- Case 2: 3 kW
- Case 3: 5 kW
- Case 4: 8 kW

(a) Parallel flow model

(b) Jet flow model

Fig. 1: The two flow models in a chamber
(a) Geometry

(b) Thermally-induced flow under natural ventilation

**Fig. 2: Chamber under natural ventilation**

**Fig. 3: Typical plume model**
The CFD package PHOENICS was used for simulating the thermally-induced air flow. The results of velocity distribution, pressure distribution and temperature distribution are shown in Figs. 4 to 6 respectively. The results of vertical velocity of the plume \( v \) are plotted in Fig. 7. Derivation of the plume model equations is listed in Appendix A. Key results related to the plume equations are:

- Velocity distribution

\[
v = \alpha \left( \frac{B}{h} \right)^{1/3} \exp\left(-\frac{65}{h^2} \frac{x^2}{h^2}\right)
\]  

where \( \alpha \) is a coefficient related to height \( h \), it is defined in Fig. 8 as:

\[
\alpha = -1.379 \ h^3 + 8.032 \ h^2 - 16.427 \ h + 17.029
\]  

and the buoyancy flux \( B \) is defined as:

\[
B = \frac{Q g}{C_p \rho T}
\]  

where \( Q \) is the heat release rate, \( g \) is the gravitational acceleration, \( C_p \) is the specific heat capacity of air, \( \rho \) is the air density and \( T \) is the absolute temperature.

From the result of this simulation, it is found that in the main part of the plume, normally \( \alpha = 4.0 \).

\[
v = 4.0 \left( \frac{B}{h} \right)^{1/3} \exp\left(-\frac{65}{h^2} \frac{x^2}{h^2}\right)
\]  

Fig. 4: Velocity distribution in central plane
Volume flux

\[ V = 0.1932B^{1/3}h^{5/3} \]  

(5)

Momentum flux

\[ M = 0.3864\rho B^{2/3}h^{4/3} \]  

(6)

Energy flux

\[ E = 0.5152\rho Bh \]  

(7)

The plume equation by Rouse [1] takes the following form:

\[ v = 4.7\left(\frac{B}{h}\right)^{1/3}\exp\left(-96.7\frac{x^2}{h^2}\right) \]  

(8)

The maximum velocity \( v_{\text{max}} \) in each horizontal level could be expressed as:

\[ v_{\text{max}} = 4.0\left(\frac{B}{h}\right)^{1/3} \]  

(12)
3. HORIZONTAL VELOCITY AT THE PLUME EDGE

From the definition of jet plume and the calculations above, on the central axis, the horizontal velocity at the plume edge is assumed to be proportional to $v_{\text{max}}$.

$$t = \frac{h}{v_{\text{max}}} = \frac{R}{u}$$

$$\frac{u}{v_{\text{max}}} = \frac{R}{h} = \tan 15^\circ$$

$$u = v_{\text{max}} \tan 15^\circ$$ \hspace{1cm} (13)

Putting equation (1) into equation (13), in the central axis, $x = 0$, the horizontal velocity distribution of the plume edge is:

$$u = \alpha \tan 15^\circ \left( \frac{B}{h} \right)^{1/3}$$ \hspace{1cm} (14)

The speed in the position of $h_0 = 0.1$ m above the floor is selected as the initial plume velocity.

Finally, the relationship between $u$ and buoyancy flux $B$ is obtained, the maximum velocity of the plume in horizontal direction is:

$$u = 0.5773 \alpha B^{1/3}$$ \hspace{1cm} (15)

The velocity at the plume edge $u$ when expressed in $\text{ms}^{-1}$ is:

$$u = 0.238 \left( \frac{Q}{1000} \right)^{1/3}$$ \hspace{1cm} (16)

The relationship between horizontal velocity of plume edge $u$ and heat release rate is shown in Fig. 10. The correlation coefficient $r^2$ is 0.9998.
Fig. 7: Relationship between vertical velocity of plume $v$ and plume radius in each layer of different height
Fig. 8: Probability distribution of height coefficient $\alpha$, correlation coefficient $r^2 = 0.9859$

Fig. 9: Comparison of plume equation with Rouse’s equation

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Fig. 8: Probability distribution of height coefficient $\alpha$, correlation coefficient $r^2 = 0.9859$

Fig. 9: Comparison of plume equation with Rouse’s equation
Fig. 10: Relationship between horizontal velocity of plume edge $u$ and heat release rate $Q$

4. PARALLEL FLOW MODEL

The plane flow model is quite well known in fluid mechanics [13]. The chamber for CFD simulation has length 4 m, width 4 m and height 3 m as in Fig. 11. There are two adiabatic walls built along two opposite sides in the perimeter, making the chamber look like a corridor.

The following assumptions were made:

- The air flow is constant, and incompressible.
- A flow with pressure difference only.
- A flow through two parallel planes.
- Turbulent mixing in the vertical direction is ignored.
- Floor, ceiling and walls of the chamber are made of adiabatic materials.

According to the above assumptions, velocity distribution can be solved by:

\[
\begin{align*}
\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{v}{\partial y} \frac{\partial w^2}{\partial y} &= 0 \\
\frac{1}{\rho} \frac{\partial P}{\partial y} &= 0
\end{align*}
\]

Taking the assumption that air flow is incompressible and $\frac{\partial w}{\partial x} = 0$, expressing in terms of the building length scale $L$, equation (17) gives:

\[
\frac{d^2 w}{dy^2} = \frac{\Delta P}{\mu L}
\]

where $\mu$ is the dynamical viscosity coefficient.

Fig. 11: Parallel flow model
\[ \Delta P = P_1 - P_2 \]  
(19)

\( P_1 \) is the air pressure on one side of the control volume and \( P_2 \) is the air pressure on the other side of the control volume [13].

\[ \Delta P = \frac{8\mu L w_{max}}{H^2} \]  
(20)

Integrating equation (18) twice by the use of the following boundary conditions:

\[
\begin{aligned}
 & y = H, \ w = U \\
 & y = 0, \ w = 0
\end{aligned}
\]

where \( U \) is the moving speed of ceiling.

The velocity distribution can be solved as:

\[ w = \frac{\Delta P}{2\mu L} \left( H h - h^2 \right) + \frac{U h}{H} \]  
(21)

The moving speed of the ceiling is 0, the flow could be considered as a pure pressure difference flow. The velocity distribution is:

\[ w = \frac{\Delta P}{2\mu L} \left( H h - h^2 \right) \]  
(22)

Putting equation (20) into equation (22):

\[ w = \frac{4w_{max}}{H^2} \left( H - h \right) h \]  
(23)

- Flux of the parallel flow

Area of the control surface is taken as \( W \cdot dy \), multiplying it with \( w \) would give the flow flux \( w \cdot W \cdot dy \). Integrating the height from 0 to \( H \):

\[ V = \int_0^H w W dy \]  
(24)

Putting in equation (21):

\[ V = \frac{W}{12 \mu L} \left( H h - h^2 \right) + \frac{U h}{H} \int_0^H \]  
(25)

The upper ceiling moving speed is 0, the flow could be considered as a pure pressure difference flow, and \( \Delta P > 0 \). The flux of the parallel flow is:

\[ V = \frac{W \Delta PH^2}{12 \mu L} \]  
(26)

Putting in equation (20):

\[ V = \frac{2HWw_{max}}{3} \]  
(27)

5. DERIVED PLUME EQUATION UNDER FORCED VENTILATION CONDITION

After studying the jet plume flow and the plane parallel flow individually, the plume equations for the two models under forced ventilation in that chamber are discussed. The plume can be described as a jet flow, the forced ventilation in a chamber can be described as a plane parallel flow. The plume equation under forced ventilation can be described as a jet flow in a parallel flow model as in Fig. 1, i.e. combining the two models together.

The chamber studied was considered as a geometry of length 4 m, width 4 m and height 3 m. There are two adiabatic walls built along two opposite sides in the perimeter. A point heat source with radius 0.1 m and height 0.1 m was located at the center of the chamber, as in Fig. 1.

Three simulations with different air flow speeds were carried out for studying the plume situation under forced ventilation condition. The results are shown in Fig. 12.

Assuming that the parallel flow has no effect on the vertical velocity of the plume, there would be no mixing and energy exchange between the plume and ambient air flow. The equations for volume, momentum flux and energy flux of the plume have the same format as those of natural ventilation. Only the position of the plume central axis would be changed by forced ventilation of the ambient air flow.

If the dynamical viscosity coefficient between the floor and ceiling is not considered, the parallel flow speed \( w \) is a constant, there would be four different situations.

When the speed of parallel flow \( w \) is zero, it means that the plume would rise without any forced ventilation. The situation of the plume without any forced ventilation is shown in Fig. 13.
The situations of the plume when the speed of parallel flow $w$ is less than, equal to and greater than the horizontal velocity of plume edge $u$ are shown in Figs. 14 to 16 respectively.

The plume central axis equation can be written as (Appendix B):

$$
\begin{align*}
\Delta \rho - \rho &= \Delta T \rho_0 w^2 t^2 \frac{Q}{4} \\
\frac{(1 - \frac{u}{T})}{T} \rho_0 HgC_p \Delta T &= \frac{2Q}{2} t^2
\end{align*}
$$

An example calculation was carried out with different velocity of parallel flow as 1 ms$^{-1}$, 2 ms$^{-1}$, 3 ms$^{-1}$ and 10 ms$^{-1}$.

The heat release rate of fire $Q$ was set to 5 kW, $T$ as 600K, $T_0$ as 300K, $g$ as 9.8 ms$^{-2}$, $H$ as 3 m, $C_p$ as 1010 J kg$^{-1}$K$^{-1}$, $\alpha$ in $v$ as 4.0 ($v_{max}$ is 1.58 ms$^{-1}$). The comparison of the plume central axis position with different velocity of parallel flow is shown in Fig. 17. The result showed that, the higher the velocity of parallel flow, the more the plume central axis is deviated from the vertical.

Another example calculation was carried out with different heat release rate of fire of 1 kW, 3 kW, 5 kW and 8 kW. The comparison of the plume central axis position with different heat release rate of fire is shown in Fig. 18. The result showed that, the bigger the heat release rate of a fire, the less the plume central axis is deviated from the vertical.

Further, in a special case, as assumed above, when the velocity of parallel flow $w$ is constant and the plume has no effects on the forced ventilation from ambient air, and the temperature inside the plume is uniform, the above equation can be simplified as:

$$
x = \frac{w}{v} h
$$

Following the settings above, with velocity of parallel flow of 1 ms$^{-1}$, the comparison of the plume central axis position in the example calculation is shown in Fig. 19. The result showed that, the simplified equation has a good fitting of the plume central axis equation, so that the simplified equation can be used in some engineering area to predict the form of a plume in a chamber.
Fig. 13: Situation of plume without any forced ventilation (the speed of parallel flow \( w \) is zero)

Fig. 14: Situation of plume when the speed of parallel flow \( w \) is less than the horizontal velocity of the plume edge \( u \)
Fig. 15: Situation of plume when the speed of parallel flow $w$ is equal to the horizontal velocity of the plume edge $u$

Fig. 16: Situation of plume when the speed of parallel flow $w$ is greater than the horizontal velocity of the plume edge $u$
Fig. 17: Comparison of the plume central axis position with different velocity of parallel flow

Fig. 18: Comparison of the plume central axis position with different heat release rate of fire
6. CONCLUSION

The plume equation under forced ventilation condition in a chamber was studied. CFD PHOENICS [14] simulations were carried out for creating plume equations under natural ventilation condition with free boundary. A comparison of the new plume equation with Rouse’s plume equation was made. The simulation results can be better described by the new plume equation after the height coefficient \( \alpha \) was introduced. Therefore, the results of the new plume equation are more precise.

The plane parallel flow was studied as an easy forced ventilation condition. Two different situations were discussed. If the dynamical viscosity coefficient between the floor and ceiling is not considered, the parallel flow speed \( w \) is constant, then there would be another four different situations: \( w = 0, \ w < u, \ w = u, \ w > u \). If the dynamical viscosity coefficient between the floor and the ceiling is considered, then the parallel speed \( w \) is not a constant, but has a distribution like plane parallel flow.

Since the parallel flow is assumed to have no effect on the vertical velocity of the plume, there would be no mixing and energy exchange between the plume and the ambient air flow. The equations for volume, momentum flux and energy flux of the plume have the same format as those of natural ventilation. Only the position of the plume central axis would be changed by forced ventilation of the ambient air flow. The position of plume central axis equation was drawn by calculation under each different condition.

NOMENCLATURE

\( A \) building area, m\(^2\)
\( a_x \) accelerate in x direction
\( a_y \) accelerate in y direction
\( B \) buoyancy flux, m\(^4\)s\(^{-4}\)
\( C_p \) specific heat capacity of air, Jkg\(^{-1}\)K\(^{-1}\)
\( E \) energy flux, kJs\(^{-1}\)
\( g \) gravity acceleration, 9.8 ms\(^{-2}\)
\( H \) height of building, m
\( h \) vertical distance above the floor, m
\( L \) length scale of building, m
\( M \) momentum flux, kgs\(^{-1}\)
\( P \) air pressure, Pa
\( Q \) heat release rate, W
\( R \) radius of plume in a specified height of \( h \), m
\( R_0 \) radius of the pool, m
\( S_0 \) vertical distance from the floor to the point heat source, m
\( T \) absolute temperature, K
\( T_0 \) absolute temperature of ambient air, K
\( U \) moving speed of ceiling, ms\(^{-1}\)
\( u \) horizontal velocity of plume edge, ms\(^{-1}\)
\( V \) volume flux, m\(^3\)s\(^{-1}\)
\( v \) vertical velocity of plume, ms\(^{-1}\)
\( v_{\text{max}} \) maximum velocity of the plume in vertical direction, ms\(^{-1}\)
\( W \) width of building, m
APPENDIX A: DERIVATION OF EQUATIONS FOR VOLUME FLUX, MOMENTUM FLUX AND ENERGY FLUX

A typical plume model is shown in Fig. 3. Taking the point heat source on the floor, \( S_0 = 0 \). Under the assumptions, the fundamental equation of vertical acceleration in two-dimensional flow is:

\[
\frac{\partial \tau}{\partial z} + \Delta B = \rho \frac{\partial^2 v}{\partial x^2}
\]

(A1)

where \( \tau \) is the vertical shear stress, proportional to the mean product of the turbulent velocity components, \( \Delta B \) is the unit buoyant force.

The equation of continuity is:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(A2)

Assuming the horizontal cross-section of the pool is round, according to the conservation law of motion, the volume flux \( V \) can be written as:

\[
V = 2\pi \int_0^\infty v_r dx
\]

(A3)

The momentum of the pool is:

\[
M = \pi \rho v_0^2 \int_0^\infty 2\pi x dx = \pi \rho v_0^2 \pi R_0^2
\]

(A4)

where \( v_0 \) is the vertical velocity in the cross-section of the pool and \( R_0 \) is the radius of the cross-section of the pool.

The momentum flux \( M \) of the plume is:

\[
M = 2\pi \int_0^\infty \rho v^2 dx
\]

(A5)

The energy flux \( E \) of the plume is:

\[
E = 2\pi \int_0^\infty \frac{\rho v^3}{2} dx
\]

(A6)

The buoyancy flux is defined as:

\[
\frac{Q_g}{C_p \rho T}
\]

(A7)

Four cases of simulations were done for studying this case. The results of velocity distribution, pressure distribution and temperature distribution
are shown in Figs. 4 to 6 respectively. The results of plume vertical velocity probability curves are plotted in Fig. 7, showing the distributions of v.

As a result, for the condition of axial symmetry model, the vertical velocity v has the following relationship with x and h [1]:

\[
\frac{v}{(B/h)^{1/3}} = f\left(\frac{x}{h}\right) \tag{A8}
\]

Suppose the velocity distribution is uniform, according to both the simulation above and the experimental results [1,11], the velocity distribution could be expressed as Gaussian distribution.

\[
v = v_{\text{max}} \exp\left(-\frac{x^2}{R^2}\right) \tag{A9}
\]

where R is the specified plume radius.

Then, the velocity distribution can be written as:

\[
v = \alpha \left(\frac{B}{h}\right)^{1/3} \exp\left(-65 \frac{x^2}{h^2}\right) \tag{A10}
\]

where \( \alpha \) is a coefficient related to height h, it is defined in Fig. 8 as:

\[
\alpha = -1.379 h^3 + 8.032 h^2 - 16.427 h + 17.029
\]

Putting this result into equations (A3), (A5) and (A6), then the following equations are obtained:

\[
V = 0.0483\alpha B^{1/3} h^{5/3} \tag{A11}
\]

Momentum flux
\[
M = 0.02415\alpha^2 \rho B^{2/3} h^{4/3} \tag{A12}
\]

Energy flux
\[
E = 0.00805\alpha^2 \rho Bh \tag{A13}
\]

From the result of this simulation, it is found that in the main part of the plume, normally \( \alpha = 4.0 \).

So equations (A10) to (A13) could be simplified as:

Velocity distribution
\[
v = 4.0 \left(\frac{B}{h}\right)^{1/3} \exp\left(-65 \frac{x^2}{h^2}\right) \tag{A14}
\]

Volume flux
\[
V = 0.1932B^{1/3} h^{5/3} \tag{A15}
\]

Momentum flux
\[
M = 0.3864\rho B^{2/3} h^{4/3} \tag{A16}
\]

Energy flux
\[
E = 0.5152\rho Bh \tag{A17}
\]

APPENDIX B: DERIVATION OF CENTRAL AXIS EQUATION

Assuming the parallel flow has no effect on the vertical velocity of the plume, then the forced ventilation of the ambient air flow has no effects on the velocity distribution in each horizontal level, so that only the position of the plume central axis would be changed.

Central axis can be seen as a link of many points. Each point was taken as forces from both horizontal direction (which is from the forced ventilation of ambient air) and vertical direction (which is from the buoyant).

Horizontal force from ventilation of ambient air:

\[
\frac{1}{2} \rho w^2 = m a_x \tag{B1}
\]

Vertical force from buoyant:

\[
(\rho_0 - \rho)Hg = m a_y \tag{B2}
\]

So that the movement of each point in central axis can be written as:

\[
x = wt + \frac{1}{2} a_x t^2 = wt + \frac{1}{2}\left(\frac{\rho_0 w^2}{2m}\right)t^2 \tag{B3}
\]

\[
h = vt + \frac{1}{2} a_y t^2 = vt + \frac{1}{2}\left(\frac{(\rho_0 - \rho)Hg}{m}\right)t^2 \tag{B4}
\]

In the plume,

\[
Q = m C_p \Delta T \tag{B5}
\]

Putting equation (B5) into equations (B3) and (B4),
In the plume,
\[
\frac{\rho}{\rho_0} = \frac{T_0}{T}
\]  
(B8)

Putting equation (B8) into equation (B7),

\[
\begin{align*}
\left\{ \begin{array}{l}
x = wt + \frac{C_p \Delta T \rho_0 w^2}{4Q} t^2 \\
h = vt + \frac{1}{2} \left( \frac{(\rho_0 - \rho) H g C_p \Delta T}{Q} \right) t^2
\end{array} \right.
\]  
(B9)

In a special case, when w is constant and the plume has no effects on the forced ventilation from ambient air, equation (B3) can be simplified to:

\[x = wt\]

and if the temperature inside the plume is uniform, equation (B4) can be simplified to:

\[h = vt\]

Then, the plume central axis equation can be written as:

\[x = \frac{w}{v} h\]  
(B10)