NOTES ON SIZING OF HORIZONTAL CEILING VENTS WITH TRADITIONAL FLOW MODEL

W.K. Chow and J. Li
Department of Building Services Engineering, The Hong Kong Polytechnic University, Hong Kong, China

(Received 5 July 2003; Accepted 20 August 2003)

ABSTRACT

Traditional flow models for smoke exhaust through horizontal ceiling vent in an atrium fire will be reviewed. There, buoyancy of the smoke layer is the driving force for extraction. Key equations on calculating the smoke exhaust rates and required vent area are derived. An atrium is taken as an example to calculate the vent areas required. Two scenarios for a fire at the atrium floor to give an axisymmetric plume; and a fire at a shop adjacent to the atrium to give a balcony spill plume are considered. It is found that a balcony spill plume will give a much higher smoke production rate and so vent with larger area is required for the same design fire in comparing with an axisymmetric plume.

1. INTRODUCTION

Natural vents are commonly installed in large atria for removing smoke [e.g. 1-4]. This is also known as static smoke exhaust system in some fire codes [e.g. 5]. Most of them are horizontal ceiling vents installed at roof. This is because many atria are located in the central core of a building, so relatively easier to allocate roof spaces than vertical walls. The driving forces for natural ventilation [e.g. 2] are stack effect due to temperature differences between indoor and outdoor; wind-induced action; and buoyancy of smoke.

In areas with low temperature difference between indoor and outdoor, stack effect is low except in tall lift shafts or staircases with high aspect ratio of height to length (or width) as demonstrated [6]. Wind-induced air flow is a transient phenomenon depending on the ambient conditions. Buoyancy of the hot smoke layer is rather strong in an atrium fire, especially at later stage of the fire. Therefore, natural vent design was based on removing smoke by taking buoyancy as the driving force. But for very tall atria, the smoke might be cooled down while moving up. Buoyancy of smoke will then be reduced to give lower extraction rate. Putting in sprinkler would also affect the system performance. All these should be considered in designing static smoke exhaust systems.

In this paper, traditional flow models [e.g. 2-4] based on buoyancy for horizontal ceiling vent was firstly reviewed. Smoke exhaust rate was studied to understand how the required vent area was estimated. Both axisymmetric plume [e.g. 12] due to a fire at the atrium floor and balcony spill plume [e.g. 13] due to a fire in a shop adjacent to the atrium will be studied.

2. FLOW ACROSS A VENT DUE TO BUOYANCY

A typical description of an elevated smoke layer in an atrium with natural vent is shown in Fig. 1a. This is the physical basis of two-layer zone models and some design guides for smoke management systems. The ceiling jet is assumed to be completely immersed in the smoke layer in most of the zone models. Circulation within the layer is not considered, giving a stagnant environment at a uniform temperature. Mixing between the smoke layer and the cool air underneath is inhibited by the density difference, and neglected in many simulations. Following analysis of Rayleigh-Taylor instability [14], lighter fluid placed above a dense fluid with an acceleration acting perpendicular towards their intersection plane will give a stable situation.

Assuming the smoke layer is effectively stagnant and thick enough to give a length scale bigger than the linear dimension of the vent. Applying Bernoulli’s theorem between points A and C [e.g. 2-4,10] with pressure $P_A$ and $P_C$ for an atrium with height $H$:

$$P_A = P_a + \frac{1}{2} \rho_g v^2 + \rho g H$$  

$$P_C = \frac{1}{2} \rho_g v^2 + P_{a0}$$

$$P_{a0} = P_0$$

$$P_C = P_0 - \rho g H$$

In the above equations, $\rho$ and $T$ are the ambient air density and temperature, $\rho_g$ and $T_g$ are the
smoke density and temperature, \( v_C \) is the outlet velocity, \( v_A \) is the inlet velocity, \( H_g \) is the smoke layer interface height, \( P_0 \) and \( P_{00} \) are the atmospheric pressures at the floor and ceiling levels respectively.

Substituting equations (3) and (4) into equation (2) gives:

\[
\frac{1}{2} \rho_g v_C^2 = P_A - P_0 + (\rho_g - \rho_a)g(H - H_g) \tag{5}
\]

From equation (1),

\[
\frac{1}{2} \rho_a v_A^2 = P_0 - P_A \tag{6}
\]

From the continuity equation:

\[
C_i \rho_A v_A A_C = C_d \rho_g v_C A_C \tag{7}
\]

where \( C_i \) and \( C_d \) are the discharge coefficients of the inlet and outlet vents respectively.

Rearranging the above equation gives:

\[
v_A = \frac{C_d \rho_A A_C}{C_i \rho_a A_A} v_C \tag{8}
\]

Eliminating \( P_0 \) and \( P_A \) from equations (5) and (6), using equation (8) gives the velocity through the outlet \( v_C \):

\[
v_C = \left[ \frac{2g(\rho_a - \rho_g)(H - H_g)}{1 + \left( \frac{\rho_g C_d^2 A_C^2}{\rho_a C_i A_A^2} \right) \rho_g} \right]^{1/2} \tag{9}
\]

Applying the ideal gas law,

\[
\rho_a T_a = \rho_g T_g \tag{10}
\]

Subtracting \( \rho_g T_a \) from both sides of the above equation,

\[
\frac{\rho_a - \rho_g}{\rho_g} = \frac{T_g - T_a}{T_g} \tag{11}
\]

Substituting equation (11) into equation (9), gives:

\[
v_C = \left[ \frac{2gT_g(T_g - T_a)(H - H_g)}{T_g + \frac{T_C d^2 A_C^2}{C_i A_A^2}} \right]^{1/2} \tag{12}
\]

The mass flow rate in the outlet \( m_e \) can be calculated by:

\[
m_e = C_d \rho_g A_C v_C \tag{13}
\]

Substituting equation (12) into the above equation gives:

\[
m_e = C_d \rho_g A_C \left[ \frac{2g(H - H_g)(T_g - T_a)T_g}{T_g + \frac{T_C d^2 A_C^2}{C_i A_A^2}} \right]^{1/2} \tag{14}
\]

### 3. REQUIRED VENTING AREA

Mass balancing of the upper smoke layer gives \( m_e \) in terms of the mass flow rate \( m_p \) at the interface height.

\[
m_e = m_p \tag{15}
\]

\( m_p \) can be expressed as:

\[
m_p = C_d \rho_a A_C \left[ \frac{2g(H - H_g)(T_g - T_a)T_g}{T_g + \frac{T_C d^2 A_C^2}{C_i A_A^2}} \right]^{1/2} \tag{16}
\]

The required ventilation area \( A_C \) is calculated by:

\[
A_C = \left( \frac{m_e}{\rho_a C_d} \right) \left[ \frac{2g(H - H_g)(T_g - T_a)T_g}{T_g + \frac{T_C d^2 A_C^2}{C_i A_A^2}} \right]^{-1/2} \tag{17}
\]

Ignoring the heat lost through the atrium, conservation of energy in the upper hot gas layer gives:

\[
m_e C_p (T_g - T_a) = m_p C_p (T_g - T_a) = \dot{Q} \tag{18}
\]

Rewriting the above equation as:
\[
\frac{T_s}{T_a} = \frac{\dot{Q}}{m_p C_p T_a} + 1
\]  

(19)

Substituting into equation (17) gives:

\[
A_c = \left( \frac{m_p}{\rho \gamma C_d} \right) \left[ \frac{\dot{Q}}{m_p C_p T_a} + 1 \right] \left[ \frac{\dot{Q}}{m_p C_p T_a} + 1 \right] \frac{C_p A_c}{C_p A_A} \frac{1}{2g(H - H_p)} \left( \frac{\dot{Q}}{m_p C_p T_a} \right)^{1/2}
\]

(20)

4. PLUME EQUATIONS

In the equation for calculating \( A_c \), expressions \( m_p \) must be determined. But the plume depends on location of the fire. Two smoke plumes are commonly encountered in an atrium:

- **Axisymmetric plume** [1-3,12] due to a fire at the atrium floor as in Fig. 1a, one of the plume models is:

  \[
  m_p = 0.071Q^{1/3}H_g^{5/3} + 0.0018Q
  \]

(21)

- **Balcony spill plume** [1-3,13] in a shop adjacent to the atrium as in Fig. 1b, with one plume model given by:

  \[
  m_p = 0.36(QW^2)^{1/3}(H_g - 0.75h)
  \]

(22)

where \( W \) is the width of the balcony and \( h \) is its height above the floor.

There had been numerous arguments [15] on selecting suitable plume equations on the above. Details on deriving those equations and validation by experiments or Computational Fluid Dynamics appeared in the literature [16,17] and will not be repeated in this paper.

Examples on calculating the required vent area are taken on two atria of height 20 m and 30 m respectively. Design fires of heat release rates \( Q \) of 1 MW, 5 MW and 10 MW are considered. Smoke layer interface height is a key design criterion as air entrainment rate of plume (and hence the smoke production rate) depends on that. This value has to be agreed while designing a natural vent.

Required vent areas are calculated for different smoke layer interface heights for both axisymmetric plume and balcony spill plume in these two atria. The balcony is of width \( W \) of 5 m and height \( H \) of 5 m above the atrium floor. Results are shown in Figs. 2 and 3.

It is observed that higher the smoke layer interface height, bigger is the required vent area. Effect of heat release rate on the vent area is not too significant for 1 MW to 10 MW. Only a slightly bigger vent might be required for bigger design fires. However, balcony spill plume will require a much bigger vent area in comparing with an axisymmetric plume, under the same design fire and same agreed smoke layer interface height. Therefore, the typical fire scenario in the atrium, i.e. assuming the fire at the atrium floor or the fire in a shop adjacent to the atrium, should be decided carefully.

---

![Fig. 1: Geometry of the problem](image-url)
5. SYSTEM PERFORMANCE

Performance of the static smoke exhaust system depends on the smoke temperature and fire size.

- Effect of smoke temperature

Expressing the smoke temperature in terms of the ambient air temperature:

$$\phi = \frac{T_s}{T_a}$$  \hspace{1cm} (23)

The vent flow equation can be rewritten as:

$$\dot{m}_e = C_d \rho_e A_c \left[ \frac{2g(H - H_i)(\phi - 1)}{\phi \left( \frac{C_d}{\phi} \frac{A_c^2}{A_s^2} \right)} \right]^{1/2}$$  \hspace{1cm} (24)

Variation of the ventilation capacity with temperature is shown in Fig. 4.
For a fixed smoke layer thickness, the mass flow rate would not always increase with the hot gas temperature. The mass flow rate would decrease with the gas temperature rising after passing through the maximum point. This critical point with maximum smoke extraction rate can be derived by taking the derivative of $m_e$ with respect to $\phi$:

$$\frac{\mathrm{d}m_e}{\mathrm{d}\phi} = \frac{C_d\rho_1 A_c}{2} \left[ \frac{2g(H - H_g)}{T_1} \right]^{1/2} \left[ \phi + \left( \frac{C_s A_c}{C_i A_1} \right)^2 \left( \frac{\phi - 1}{\phi + 1} \right)^{1/2} \right]$$

And then setting

$$\frac{\mathrm{d}m_e}{\mathrm{d}\phi} = 0$$

Physically, the system performance will not be affected by smoke temperature, giving:

$$\phi = 1 + \left( \frac{C_s A_c}{C_i A_1} \right)^2 + 1 \right)^{1/2}$$

Provided that the area of inlets is large compared to the area of the vents, i.e. $A_A >> A_C$,

$$\phi \approx 2$$

This gives:

$$T_g = 2T_a$$

and so

$$\Delta T = T_g - T_a \approx T_a$$

Under this condition, $m_e$ takes the maximum value $m_{\text{max}}$.

- Effects of fire size on the mass flow rate through a vent

Provided that the inlets area is much larger than the vents area ($A_A >> A_C$), equation (16) can be written as:

$$m_e \approx C_d\rho_1 A_c \left[ 2g(H - H_g) \left( \frac{T_1}{T_1} - 1 \right) \right]^{1/2} \left( \frac{T_1}{T_1} \right)^{1/2}$$

Let

$$A^* = C_d\rho_1 A_c \cdot (2g)^{1/2}$$

Substituting into equation (27) with equation (19) gives:

$$Qm_e^{1/2} + m_e^{3/2} C_p T_a = A^* (H - H_g)^{1/2} \frac{Q}{2} (C_p T_a)^{1/2}$$

(29)

For $m_e = m_p$,

$$Qm_p^{1/2} + m_p^{3/2} C_p T_a = A^* (H - H_g)^{1/2} \frac{Q}{2} (C_p T_a)^{1/2}$$

(30)
Note that the mass flow rate $m_p$ of the plume at the interface height is a function of $Q$ and $H_g$, given by either equation (21) or (22) for different configurations. The smoke layer interface height in an atrium under a certain fire can be estimated by equation (30) once the vent area is known.

6. CONCLUSION

In this paper, traditional vent flow model for natural vent was reviewed. Buoyancy is the driving force and derivation was based on applying Bernoulli’s theorem. The required vent area is estimated from the derived smoke exhaust rate.

Examples on two atria of heights 20 m and 30 m were taken to calculate the required vent areas for 1 MW, 5 MW and 10 MW fires. Both axisymmetric plume and balcony spill plume were considered.

It is found that heat release rate would not affect the vent area significantly. Perhaps, it is not necessary to spend time on arguing the design fire size, if static smoke exhaust system is to be provided.

However, a balcony spill plume due to a fire in a shop adjacent to the atrium would give higher air entrainment rate, and so bigger vent area is required than the same fire for an axisymmetric plume. The fire scenario must be selected carefully on designing static smoke exhaust system. Both the smoke layer interface height and the fire scenario to give an axisymmetric plume or a balcony spill plume must be discussed and agreed. The smoke layer should be kept above the occupied zone in order not to emit high radiative heat flux for occupants and firefighters staying below.

ACKNOWLEDGEMENT

The project is funded jointly by an RGC grant and The Hong Kong Polytechnic University.

REFERENCES

5. Fire Services Department, Codes of Practice for Minimum Fire Service Installations and Equipment and Inspection and Testing of Installations and Equipment, Hong Kong (1994).